A Straightforward ICCG Convergence Method for Simulation of Multi-loop and FE Model of Electric Machines and Power Electronics Systems

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Abstract — Now electric machines integrate with power electronics to form inseparable systems in a lot of applications for high performance. For such systems, two nonlinearity exist at the same time, which makes simulation time-consuming: the magnetic nonlinearity of iron core and the circuit nonlinearity caused by power electronics devices. In this paper, the multiloop model combined with FE model of a AC-DC synchronous generator, as one example of electric machine and with power electronics system, is set up. FE method is used for magnetic nonlinearity, and variable step variable topology simulation for circuit nonlinearity. In order to improve the simulation speed, Incomplete Cholesky-Conjugate Gradient method (ICCG) is used to solve the state equation. However, when the state of power electronics devices changes, the convergence difficulty occurs. So a straightforward approach to achieve convergence of simulation is proposed. At last, the simulation results compare with the experiments.

I. INTRODUCTION

Now power electronics are widely used in electrical machine system, which promotes great flexibility of electrical machine control, but at the same time fast and frequent switching of power electronics devices cause some problems, such as harmonics. So the research on electrical machine and power electronics system is necessary.

For such systems, two nonlinearity exist at the same time, which makes simulation time-consuming: the magnetic nonlinearity of iron core and the circuit nonlinearity caused by power electronics devices.

Two effective way to analyze and simulate such complex systems are finite element method for magnetic nonlinearity, and variable step variable topology simulation for circuit nonlinearity. Now the circuit model combined with FE model is effective and popular for detail analysis of such systems[1]-[5].

But because of the nonlinear characteristic of magnetic path, one iteration for every step is needed and the simulation is time-consuming. If the power electronics devices are involved in, the situation would be worsen.

In order to improve the simulation speed, ICCG is used to solve the state equation[6]-[8]. However, when the state of power electronics devices changes, the convergence difficulty occurs[7][8]. So a straightforward approach to achieve convergence of simulation is proposed, and a AC-DC synchronous generator including four rectifier bridges is used as an example, as shown in Fig.1. At last, the simulation results are compared with the experiments.

Fig. 1. The AC-DC generator under investigation

II. MATHEMATIC MODEL

A. Multi-loop model of generators

For multi-loop model, each winding is regarded as a circuit, as is each short-circuited loop in the damper cage. The multi-loop model of generators can be expressed as:

 $U_F = -L_L pI_F - p\Psi_M - R_F I_F$ where the differential operator $p = d/dt$, L_L is the leakage inductance matrix of end winding. The flux linkage Ψ_M is related to the saturation of iron core and should be calculated by FE model.

B. FE Model of Generator

The FE model of generators can be expressed as:

$$
\mathbf{S} \cdot \mathbf{A}_F = \mathbf{C} \cdot \mathbf{I}_F \tag{2}
$$

where A_F is the unknown vector of vector potentials of nodes. S is the coefficient matrix formed by the element analysis of FEM. C is the connection matrix representing the relationships between the currents and the nodes.

The Ψ_M in (1) is $\Psi_M = 2Pl_{ef}C^T \cdot A_F$, where *P* is pole pair number, l_{ef} is the length of iron core.

C. System Model

Combining the equations of generator and loads, a system circuit equation is gotten:

$$
U = -L pI - p\Psi - RI \tag{3}
$$

where in the \mathbf{R}, \mathbf{L} and $\mathbf{I}, \mathbf{U}, \mathbf{\Psi}$, the quantities associating with the AC and DC loads are appended.

After combined (3) and (2), and discretized, the system model can be obtained as:

$$
\begin{bmatrix} S & C' \\ C'^T & -\frac{\Delta t \cdot R}{4Pl_{cf}} - \frac{L}{2Pl_{cf}} \end{bmatrix} \begin{bmatrix} A_{n+1} \\ -I_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ C^T & -\frac{\Delta t \cdot R}{4Pl_{cf}} + \frac{L}{2Pl_{cf}} \end{bmatrix} \begin{bmatrix} A_n \\ I_n \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\Delta t \cdot U_n}{2Pl_{cf}} \end{bmatrix} (4)
$$

III. CONNECTION TRANSFORMATION

Equation (4) is only the model of branches, not the actual loops and can not be solved directly. So one connection transformation is needed to form independent loops model, which is decided by the operating load condition, and the state of diode. The connection can be represented by a connection transformation matrix *T*. The connection transformation matrix is important and convenient for the simulation of electric machine and power electronics system. It should be noted that once a power electronics device, such as a diode in this paper, switches on or off, the connection transformation matrix *T* must be re-determined to represent the variable topology.

After founded the connection transformation matrix, the voltages and currents can be transformed as $U' = TU$, $I = T^T I'$. Then, the actual equation can be expressed as:

$$
\begin{bmatrix}\nS & C'T^T \\
TC'^T & T(-\frac{\Delta t \cdot R}{4Pl_{\text{cf}}}-\frac{L}{2Pl_{\text{cf}}})T^T\n\end{bmatrix}\n\begin{bmatrix}\nA_{n+1} \\
-I_{n+1}\n\end{bmatrix} =\n\begin{bmatrix}\n\theta & \cdot \\
TC'^T & T(-\frac{\Delta t \cdot R}{4Pl_{\text{cf}}}+\frac{L}{2Pl_{\text{cf}}})T^T\n\end{bmatrix}\n\begin{bmatrix}\nA_n \\
I_n\n\end{bmatrix} + \begin{bmatrix}\n\theta & \cdot \\
T(-\frac{\Delta t \cdot U_n}{2Pl_{\text{cf}}})\n\end{bmatrix}
$$
\n(5)

The multi-loop model and FE model are assembled into one equation so that it eliminates the iteration between these two models while they are solved separately.

IV. SIMULATION

While a switching action takes place, the elements in the matrix must be modified. In order to find out the commutation instant accurately, a variable step method is used to simulate. If there is no commutation within the next step, step forward keeping the current step. If a commutation occurs within the next step, cut half the step until that the step reaches the minimal step or no commutation occurs.

Because of the nonlinearity of iron core, Newton-Raphson iteration method is used to solve the nonlinear equation (5).

V. ICCG METHOD AND GAUSS ELIMINATION METHOD

Originally, Gauss elimination method is used to solve the state equation, the process goes smoothly and the results are consistent with the experiments. But because of magnetic nonlinearity of iron core and the circuit nonlinearity of diode, two iteration is included: Newton-Raphson iteration and variable step iteration. So the simulation process is very time-consuming.

To shorten the simulation time, as many published literature, the ICCG method is used for solution of the state

equations. When the topology of circuit is not changed, the effect is evident. The comparison of simulation time by these two methods will be given in the full paper.

VI. CONVERGENCE METHOD FOR ICCG

When ICCG is used to simulate the variable topology structure circuit, one difficulty occur. Once the state of one diode changes, the connection transformation matrix is modified, then the simulation is not convergent and process can not proceed. It is interesting that this difficulty does not occur for Gauss elimination method.

In order to keep the advantage of shorten simulation time of ICCG and achieve the numerical convergence, a straightforward approach, by combining the ICCC method and Gauss elimination method, is proposed in this paper. The ICCG method is used to the solve equations in the simulation process, except just one simulation step after the topology structure of circuit changes. Once the topology structure of circuit changes, the Gauss elimination is adopted to simulate one step, after that ICCG method can be used immediately. By this way, the numerical instability would not occur.

VII. COMPARISON OF SIMULATION AND EXPERIMENT

At last, the validation of this simple approach is verified by the comparison of performance simulation and experiments.

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